Physics 106c
Problem set number 10
Due Wednesday, April 14, 1999

Notes about course:

- Reminder: Old homeworks may be retrieved, and new homeworks turned in, in room 335 Lauritsen.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand. Don't look at solutions from earlier year(s), though you can ask people who took it before for advice.
- Text: *Classical Electrodynamics* (third edition), by J. David Jackson.
- The graders are the same as last quarter:

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- Late policy: If you need to turn an assignment in late, get an OK from either me or from one of the TAs, and put a note on the late assignment saying that you got the OK. You should normally get the OK before the due date, but I'll be flexible if you are ill, or if there is an unexpected emergency.

  If you turn in a late assignment without an OK, it is up to the discretion and mercy of the grader. Probably they'll subtract credit, or, if it is really late, they may not accept it at all.
- Web page: I'll try to put problem sets on a web page, with URL: `http://www.cithep.caltech.edu/~fcp/ph106/`

Reading: Jackson chapter 11, Sections 1-10, if you haven't finished it. Next, we'll go to chapter 4, Sections 3-7.

53. Jackson problem 11.19.
54. Jackson problem 11.20.
55. Jackson problem 11.22.
56. In addition to the momentum and energy, there are several more specialized kinematical variables which are handy in describing particle interactions. In the following, we suppose z is a "special" chosen spatial direction. In practice, it is usually useful to pick this direction to be along the direction of one of the incoming particles in a colinear two-body collision process, or sub-process. In such a case, we refer to this direction as the "longitudinal" direction, and the perpendicular directions as "transverse".

- The rapidity, y, of a particle is an alternative to p_z as a measure of the particle's longitudinal motion. It is defined as:

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right). \tag{1}$$

- Pseudorapidity: The pseudorapidity, η, of a particle is a convenient approximation to the rapidity, valid at high energies and transverse momenta not too near the maximum. It is defined according to:

$$\eta = -\ln \tan \theta/2, \tag{2}$$

  where θ is the angle between the particle and the z-axis.

(a) Show that rapidity differences are invariant with respect to longitudinal Lorentz transformations. Therefore, all longitudinal velocity boosts are pure translations in rapidity, with the value of the translation dependent only on the boost velocity.

(b) Show that, in the appropriate regime, $\eta \sim y$. Give the appropriate kinematic region.

57. The η meson has spin 0. Thirty-nine per cent of the time it decays to two photons. The mass of the eta is 547 MeV. Imagine doing an experiment where we observe $\eta \to \gamma\gamma$ decays, in which we measure the energies of the decay photons. You may assume a "perfect detector", covering 4π solid angle with infinitely good resolution at all relevant photon energies.

    (a) If the η is produced exactly at rest (in the detector frame), what is the angular distribution, $\frac{dN}{d\Omega}$, of the observed photons? What is the energy distribution, $\frac{dN}{dE_\gamma}$?

    (b) If the η is moving with momentum p_η, what is the energy distribution $\frac{dN}{dE_\gamma}$?

58. Optional bonus question: We consider the process $X \to 1 + 2 + 3$, where initial state X goes to three particles, 1,2, and 3. The differential decay rate (if X is a particle) may be written:

$$d\Gamma = \frac{1}{2m_X} |\mathcal{M}|^2 dLIPS,$$

    in the X rest frame, with

$$dLIPS = (2\pi)^4 \delta^{(4)}(p_X - p_1 - p_2 - p_3) \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}.$$

    (a) "$LIPS$" stands for "Lorentz Invariant Phase Space". Show that it is appropriately named, *i.e.*, that it is Lorentz invariant.

(b) We suppose we do an experiment where we measure events in the above decay. Show that the density of points on a scatterplot of E_1 *vs.* E_2 is proportional to

$$|\mathcal{M}(E_1, E_2)|^2 dE_1 dE_2$$

In particular, notice that if the invariant matrix element is a constant, then the graph has a uniform density of points within the kinematic boundaries.

(c) If m_{ij} is the invariant mass of particles i and j (*i.e.*, $m_{ij}^2 = (p_i + p_j)^2$), show that an equivalent approach is to make a graph of m_{13}^2 *vs.* m_{23}^2 (*i.e.*, a uniform density in $dE_1 dE_2 \Leftrightarrow$ uniform density in $dm_{13}^2 dm_{23}^2$).

(d) Make a careful picture showing the kinematic boundary for the three-body decay plot with axes m_{13}^2 and m_{23}^2 for the decay $J/\psi \rightarrow \gamma\gamma\gamma$. Study the attached three-body decay plot (Fig. 2) and discuss the structure. You may wish to understand the boundary drawn – note that the bottom boundary line depends on the experimental cuts.

(e) You determined the three body decay plot kinematic boundary for a 3 photon decay. Now find the kinematic boundary for the general case, of a particle of mass M decaying to three particles with masses m_1, m_2, and m_3. Pick some set of (m_i/M), $i = 1, 2, 3$ (different from above) and make a graph in the $(m_{13}/M)^2, (m_{23}/M)^2$ plane showing what the boundary looks like.