Physics 125c
Problem set number 8
Due Wednesday, May 26, 2004

Notes about course:

- There is a web page for this course, which should be referred to for the most up-to-date information. The URL:
  http://www.hep.caltech.edu/~fcp/ph125/

READING: Read sections 1-5 of the "Density Matrix Formalism" course note.

PROBLEMS:

28. Let us pursue further the application of our discussion on electromagnetic interactions in atomic physics. Consider in particular the common case in which the wavelength of light emitted by an atom is much larger than the atomic dimensions. Recall that the transition rate for spontaneous emission from atomic state $|n\rangle$ to state $|0\rangle$ is:

$$\Gamma_{\mathbf{k}\boldsymbol{\epsilon}}(\mathrm{em}; n \to 0) = \frac{4\pi^2 q^2}{\omega V}\delta(E_n - E_0 - \omega)|\langle 0|\hat{\mathbf{j}}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^*|n\rangle|^2. \tag{22}$$

    (a) Show that, to a good approximation we may evaluate the atomic transition matrix element according to:

$$\langle 0|\hat{\mathbf{j}}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^*|n\rangle = -i\omega\langle 0|\mathbf{x}|n\rangle \cdot \boldsymbol{\epsilon}^*, \tag{23}$$

    where $\mathbf{x} = \sum_i \mathbf{x}_i$, which may be referred to as the dipole moment operator (multiply it times q, and you have the electric dipole operator). This approximation is known as the electric dipole approximation. [Hint: Evaluate the commutator of $\mathbf{x}$ with H_0.]

    (b) Using your result, estimate the lifetime of the $^1P_1$ state of atomic hydrogen (in the lowest radial state), as it decays to the $^1S_0$ ground state.

29. Do exercise 1 of the "Density Matrix Formalism" course note.

30. Do exercise 2 of the "Density Matrix Formalism" course note.

31. Do exercise 3 of the "Density Matrix Formalism" course note.