PROBLEMS:

13. In the notes we derived the optical theorem assuming that we had a “symmetric central force”. Show that this assumption is unnecessary. Hint: This is trivial, except for one piece of the assumption which you will have to retain.

**Solution:** Start with the step prior to making the assumption in the notes:

$$-\frac{i}{2\pi} \frac{\delta(p' - p'')}{p'} \left[ f(p', p'') - f^*(p'', p') \right] = \frac{\delta(p' - p'')}{4\pi^2} \int_{(4\pi)} d\Omega_u f(p', q) f^*(p'', q).$$  \hspace{1cm} (10)

Note that we must have $p' = p'' = q \equiv p$. Thus, write:

$$-\frac{i}{p} \left[ f(p'u', pu'') - f^*(pu'', pu') \right] = \frac{1}{2\pi} \int_{(4\pi)} d\Omega_u f(p'u', pu) f^*(pu'', pu).$$  \hspace{1cm} (11)

Now consider forward scattering: $u'' = u'$:

$$-\frac{i}{p} \left[ f(p'u', pu') - f^*(pu', pu') \right] = \frac{1}{2\pi} \int_{(4\pi)} d\Omega_u f(p'u', pu) f^*(pu', pu).$$  \hspace{1cm} (12)

With the assumption that $f(p'u', pu) = f(pu, pu')$, we immediately see that we have once again the optical theorem:

$$\sigma_T(p) = \frac{4\pi}{p} \Im f(p; 1).$$  \hspace{1cm} (13)

Note that the assumption we retained was that the scattering amplitude is invariant (up to a phase) under interchange of incoming and outgoing directions.

14. Do exercise 7 of the “Scattering” course note.

15. Do exercise 8 of the “Scattering” course note.

16. Do exercise 9 of the “Scattering” course note.