Physics 129a
Problem Set Number 1
Due Wednesday, October 7, 2009

Reading: Sections 1–3 of the note (posted on the course web page) on "Integral Equations". This provides an introduction to integral equations and discusses integral transforms, in particular the Fourier and Laplace transforms. I expect that the material on integral transforms, perhaps excepting Laplace's method, will be familiar to many of you, but I would like to hear from you if not (or even if so)!

Please turn assignments in either to class or to the appropriate TA's mail slot on the fourth floor of Downs. Chan Youn Park will grade the odd-numbered assignments, and Hee-joong Chung the even ones.

As the point of this course is to learn techniques as well as the theory behind them, I urge you to avoid look-up tables (e.g., of integrals). If you do feel the need to resort to tables, however, be sure to state your source.

1. Exercise 4 in the Integral Equations course note.

2. Exercise 7 in the Integral Equations course note. We'll make some comments in class about how the solution is greatly simplified with symmetry argments.

3. Exercise 8 in the Integral Equations course note.

4. The radioactive decay of a nucleus is a random process in which the probability of a decay in time interval $(t, t+dt)$ is independent of t, if the decay has not already occurred. This leads to the familiar exponential decay law (as you may wish to convince yourself): If at time t, there are $N(t)$ nuclei, then the rate of decays is proportional to $N(t)$:

$$\frac{dN}{dt}(t) = -\lambda N(t).$$

Integrating, we find $N(t)$:

$$N(t) = N(0)e^{-\lambda t}.$$

In practice, radioactive decays often occur in long chains. For (a simplified) example, ^{238}U decays via α-emission to ^{234}Th with a half-life of 4.5×10^9 y; ^{234}Th decays in a subchain with two β emissions to ^{234}U with a half-life of 24 d; ^{234}U decays via α-emission to ^{230}Th with a 2.4×10^6 y half-life; etc. We may use the method of Laplace transforms to determine how the abundance of any species of nucleus in such a chain evolves with time.

Thus, suppose that we have a decay chain $A \rightarrow B \rightarrow C \rightarrow D$, where D is stable, and the decay rates for A, B, and C are λ_A, λ_B, and λ_C, respectively. Suppose $N_B(0) = 0$. Determine $N_C(t)$ as a function of the rates and the initial abundances $N_A(0)$ and $N_C(0)$.

You are supposed to approach this problem by setting up a system of differential equations for the abundances, and then using Laplace transforms to solve the equations. Note, in setting up your differential equations, that the rate of change in abundance for an intermediate nucleus in the chain gets a contribution from the nucleus decaying into it as well as from its own decay rate.