Physics 129a
Problem Set Number 7
Due Tuesday, November 17, 2009

Reading: Finish reading the course note on "Linear Differential Equations".

29. The approach of a delta-function "source" in solving an inhomogeneous differential equation: Exercise 4 in the Introduction to Distributions course note. Note that even though the boundary conditions are specified at finite x, the specified range of x over the whole real axis still holds.

30. Follow-up on introduction in class of the Sturm-Liouville equation: Exercise 1 in the Linear Differential Equations course note.

31. Consider the differential equation:

$$x^2\frac{d^2u}{dx^2} + 2x\frac{du}{dx} - \ell(\ell+1)u = \delta(x-y), where \tag{5}$$

where $x \in (0, \infty)$, $y > 0$, and $\ell > 0$ is an integer. Solve this equation for the homogeneous boundary conditions $u(0) = u(\infty) = 0$

32. Hypergeometric function: Exercise 8 in the Linear Differential Equations note. Note that $u(x) =_2 F_1(a, b; c; x)$ is a solution to the hypergeometric equation (133) in the note on Linear Differential Equations. It is the solution which remains finite as $x \to 0$. You may wish to verify that the following series form is a solution to the hypergeometric equation:

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!}, \tag{6}$$

as well as the integral representation:

$${}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 dy y^{b-1}(1-y)^{c-b-1}(1-yx)^{-a}. \tag{7}$$

We require here that the real parts of a and b are positive, and that the real part of a is larger than the real part of b.

33. Orthogonal polynomial recurrence relation: Exercise 5 in the Linear Differential Equations note.