Reading: Read chapter 8 in the text, on the electrodynamics of quarks and hadrons.

30. Let us prove some of the theorems that are so handy in calculating Feynman graph amplitudes. You may use the anticommutation relations for the gamma matrices:

\[ \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}, \]

as well as whatever theorems in matrix theory you know.

(a) Prove that \( \langle a | b \rangle + \langle b | a \rangle = 2a \cdot b. \)

(b) Show that \( \gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\mu = 4g^{\alpha\beta} \), and hence that \( \gamma^\mu \langle a | b \rangle \gamma^\mu = 4a \cdot b. \)

(c) Show that the trace of the product of an odd number of gamma matrices is zero.

(d) Show that

\[ \text{Tr}(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) = 4\left(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}\right), \]

and hence that

\[ \text{Tr}(\langle a | b \rangle \langle c | d \rangle) = 4(a \cdot b \cdot c \cdot d - a \cdot c \cdot b \cdot d + a \cdot d \cdot b \cdot c). \]

(e) Show that \( \text{Tr}(\gamma^5) = 0. \)

31. Problem 7.21 in text.

The next two problems lead you through your first Feynman graph calculation:

32. Problem 8.1 in text.

33. Problem 8.2 in text.

34. Problem 8.16 in text. This should be easy.