92. Consider a beamline with $p = 100$ GeV protons, kaons, and pions. We wish to tag the beam particles according to their identity. A standard technique for this is to use Cherenkov radiation. If we try to separate all three with a single device, a threshold counter is not sufficient, so we might try an imaging device in which we measure the angle of the Cherenkov emission. Let us try to design such a device. We use a low density radiator (helium gas at some pressure and temperature, say), so that the amount of material is small – we plan to use the tagged beam in some fixed-target experiment or test beam, and so that fractional differences in the Cherenkov angle are large enough. However, with a low density radiator, we must have enough of it to get sufficient light, implying a long region of emission. It is then necessary to find a focusing system which takes out the point of emission. Such a scheme uses a spherical mirror, as shown below:
Design such a particle identifier to separate $p$, $K$, and $\pi$ at 100 GeV momentum. Use helium at a temperature and pressure which you specify (please don’t make a bomb). Assume that you can measure the position of a photon to 0.2 cm.

93. This problem is motivated by the two- and three-pion decay neutral kaon decay phenomenology. Show that the two pions in a $K^0 \rightarrow \pi\pi$ decay must be in a $CP$ even state, for both $\pi^0\pi^0$ and $\pi^+\pi^-$. Now consider decays to $K^0$ to three pions, in relative $S$-wave. What $CP$ states are allowed?

94. According to the 2004 Review of Particle Properties, the $Z$ has an “invisible” width of 499.0±1.5 MeV.

   (a) How is this measured?

   (b) Compare this result with the expectation of the standard model. What can you say about the number of generations?

95. Carrying further the discussion begun in problem 31, now consider a two regenerator experimental setup, in which there are two targets, separated by a distance $d$. Obtain a formula for the intensity of $2\pi$ decays after the second regenerator, as a function of $d$. Show how this setup may be used to accurately measure the $K_S$, $K_L$ mass difference. You may assume that the thickness of a target is small compared with $d$, and that the two targets are identical.