Physics 231b  
Problem Set Number 11  
Due Wednesday, April 12, 2000

Note: Some problems may be “review” for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write “been there, done that”, or suitable equivalent, for that problem, and I’ll give you credit.

52. In class, we asserted that the most general form for the proton electromagnetic current in ep elastic scattering is:

\[ J^\mu(x) = e\bar{u}(p') \left[ F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)i\sigma^{\mu\nu}q_\nu \right] u(p)e^{i(p'-p) \cdot x}, \]

where \( p \) is the proton’s initial 4-momentum, \( p' \) its final 4-momentum, and \( q = p' - p \). Justify this assertion, i.e., explain why the other possible Lorentz 4-vectors constructable from \( p, p' \), and the gamma matrices don’t appear. This is a typical sort of procedure – write down the most general thing you can, then figure out what terms are really independent, and what contributions really may be present given constraints such as current conservation. The “form factors” \( F_1 \) and \( F_2 \) parameterize our remaining ignorance of the proton structure.

53. The Breit Frame: Let us try to get some further intuition concerning the elastic from factors \( G_E \) and \( G_M \).

(a) Show that the proton current of the previous problem can be rewritten in the form:

\[ J^\mu(x) = e\bar{u}(p') \left[ (F_1 + \kappa F_2)\gamma^\mu - \frac{(p^\mu + p'^\mu)}{2M} \kappa F_2 \right] u(p)e^{i(p'-p) \cdot x}, \]

(b) The Breit frame, or “brick wall” frame is defined as the frame in which \( p' = -p \). Choose \( p \) to be along the z axis. Compute the proton current in this frame for the different possible helicity states, and hence relate the charge density \( \rho \), and current density \( \mathbf{J} \) to \( G_E \) and \( G_M \).
54. Let’s think about some of the useful kinematic variables introduced in class, in our discussion of deep inelastic scattering. Recall, in lepton-hadron scattering:

\[-Q^2 \equiv q^2 = (k - k')^2,\]

where \(k\) and \(k'\) are the initial and final lepton momenta, respectively. Also,

\[\nu \equiv \frac{p \cdot q}{M},\]

where \(p\) is the initial hadron momentum, and \(M\) is its mass. The pair \((Q^2, \nu)\) is a suitable choice of kinematic invariants for describing the scattering. However, we could also choose the dimensionless pair \((x, y)\), where:

\[x \equiv \frac{Q^2}{2 p \cdot q}, \quad y \equiv \frac{p \cdot q}{p \cdot k}.\]

(a) Determine the kinematically allowed regime in \(x\) and \(y\), for \(\mu p \to \mu X\) scattering. You may use the relativistic limit as desired.

(b) Make a graph of the mapping of lines of constant \(x\) and of constant \(y\) on the \((Q^2, \nu)\) plane.

55. In class, we wrote down the high energy differential cross section, in the laboratory frame, for inelastic \(e p \to e X\) scattering:

\[
\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[ W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} \right].
\]

Show that this may be rewritten in the frame-invariant form:

\[
M \nu_{\text{max}} \frac{d\sigma}{dx dy} = \frac{2\pi \alpha^2}{x^2 y^2} \left\{ x y^2 F_1 + \left[ (1 - y) - \frac{M x y}{2\nu_{\text{max}}} \right] F_2 \right\}.
\]

56. When I compared the interaction time with the time scale for parton interaction within the proton wavefunction in class, I made a simple estimate for the latter timescale. In particular, I argued that this time scale should be of the order of the size of the proton. We may also estimate this timescale from a somewhat different perspective: We consider the proton wavefunction to consist of a superposition of virtual states, described by a parton (the “struck” parton, in particular) carrying momentum fraction \(x\), and the remainder of the partons carrying fraction
1 − x. As we noted in class, except in the ∞-momentum frame, such
a configuration violates energy conservation, because of the finite mass
of the partons, and because of their finite transverse momenta within
the proton. This, such a state must have a lifetime consistent with the
“uncertainty relation” $\Delta E \Delta t \gtrsim 1$.

Let us suppose that we have a proton of momentum $|\vec{p}| = p \gg M$,
where $M$ is the proton mass. Imagine a configuration where a parton
with rest mass $m_1$ and transverse momentum $q_{\perp 1}$ carries a fraction $x$ of
the parton’s momentum. For convenience, lump whatever else there is
into a “parton” of mass $m_2$, transverse momentum $q_{\perp 2}$ and longitudinal
momentum fraction $(1 − x)$. Show that an estimate for the lifetime of
a virtual state is given by:

$$T \simeq \frac{2P}{m_{\perp 1}^2/x + m_{\perp 2}^2/(1 − x) − M^2}$$

where the “transverse masses” $m_\perp$ are given by:

$$m_{\perp 1}^2 = m_1^2 + q_{\perp 1}^2$$

$$m_{\perp 2}^2 = m_2^2 + q_{\perp 2}^2$$

Note that this estimate is not in disagreement (see if you can convince
yourself) with the estimate given in class, except possibly for the very
short-lived limiting cases $x \to 0$ and $x \to 1$. 

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