Physics 195a
Course Notes
Solving the Schrödinger Equation: Resolvents
Solutions to Exercises
021209 F. Porter

1 Exercises

1. Prove identities (12) and (13).

2. Prove the power series expansion for resolvent $G(z)$ (Eqn. 2):

$$G(z) = G(z_0)\sum_{n=0}^{\infty}\left[(z-z_0)G(z_0)\right]^n .$$

You may wish to attempt to do this either "directly", or via iteration on the identity of Eqn. 11.

3. Prove the result in Eqn. **??**.

4. Let's consider once again the Hamiltonian

$$H = -\frac{1}{2m}\frac{d^2}{dx^2}, \tag{1}$$

but now in configuration space $x \in [a,b]$ ("infinite square well").

(a) Construct the Green's function, $G(x,y;z)$ for this problem.
Please do not look at this solution until you have turned in problem set number 8
Solution: To construct the Green's function, we look for solutions to:

$$Hu(x) = -\frac{1}{2m}\frac{d^2}{dx^2}u(x;z) = zu(x;z). \tag{2}$$

The solutions may be expressed in the form

$$u(x;z) = A\sin\rho(x+\alpha), \tag{3}$$

where $\rho \equiv \sqrt{2mz}$. The left and right solutions, giving the boundary conditions $u(a) = u(b) = 0$ are thus:

$$u_L(x;z) = A\sin\rho(x-a) \tag{4}$$
$$u_R(x;z) = B\sin\rho(b-x). \tag{5}$$

The Green's function is:

$$G(x,y;z) \equiv \frac{2m}{W(z)}\left[u_L(x;z)u_R(y;z)\theta(y-x) + u_L(y;z)u_R(x;z)\theta(x-y)\right], \tag{6}$$

where the Wronskian is

$$W(z) \equiv u_L'(x;z)u_R(x;z) - u_L(x;z)u_R'(x;z). \tag{7}$$

Since the Wronskian is independent of x, we pick a convenient place to evaluate it, $x=b$:

$$W(z) = AB\rho\sin\rho(b-a). \tag{8}$$

Hence, the Green's function is

$$\begin{aligned} G(x,y;z) &= \frac{2m}{\rho\sin\rho(b-a)} \\ &\quad\times\Big[\sin\rho(x-a)\sin\rho(b-y)\theta(y-x) \\ &\qquad+\sin\rho(y-a)\sin\rho(b-x)\theta(x-y)\Big]. \end{aligned} \tag{9}$$

(b) From your answer to part (a), determine the spectrum of H.

Solution: We first remark that G is regular at $\rho=0$, hence at $z=0$. The poles appear at:

$$\sqrt{2mz}(b-a) = k\pi, \quad k=\pm1,\pm2,\ldots \tag{10}$$

That is, the eigenvalues of H are:

$$\omega_k = \frac{\pi^2k^2}{2m(b-a)^2}, \quad k=1,2,\ldots \tag{11}$$